Deep Learning for Vision & Language

Machine Learning III: Softmax Classifier / Multi-layer Perceptrons

About the class

- COMP 646: Deep Learning for Vision and Language
- Instructor: **Vicente** Ordóñez (Vicente Ordóñez Román)
- Website: https://www.cs.rice.edu/~vo9/deep-vislang
- Location: Herzstein Hall 210
- Times: Tuesdays and Thursdays
  from 4pm to 5:15pm
- Office Hours: Tuesdays 10am to 11am (DH3098)
- Teaching Assistants: **Arnold, Jefferson, Sangwon, Gaotian**
- Discussion Forum: Piazza (Sign-up Link on Rice Canvas and Class Website)

Teaching Assistants (TAs)

Jefferson Hernandez

Mondays 2:30pm
DH 3036

Sangwon Seo

Wednesdays 10am
DH 3002

Gaotian Wang

Wednesdays 3pm
DH 3036

Arnold Kazadi

Thursdays 11am
DH 3036

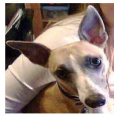
Assignment 1

- Assignment 1 is released and is available on the class website.

Supervised Learning - Classification

Training Data

cat



dog

cat

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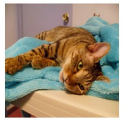
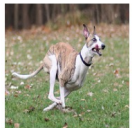
bear

Test Data





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Supervised Learning - Classification

Training Data

$x_1 = [$  $]$ $y_1 = [$cat $]$

$x_2 = [$  $]$ $y_2 = [$dog $]$

$x_3 = [$  $]$ $y_3 = [$cat $]$

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.
.

$x_n = [$  $]$ $y_n = [$bear $]$

Supervised Learning - Classification

Training Data

inputs

targets / labels / ground truth

predictions

$x_1 = [x_{11} \quad x_{12} \quad x_{13} \quad x_{14}]$ $y_1 = 1$ $\hat{y}_1 = 1$

$x_2 = [x_{21} \quad x_{22} \quad x_{23} \quad x_{24}]$ $y_2 = 2$ $\hat{y}_2 = 2$

$x_3 = [x_{31} \quad x_{32} \quad x_{33} \quad x_{34}]$ $y_3 = 1$ $\hat{y}_3 = 2$

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$x_n = [x_{n1} \quad x_{n2} \quad x_{n3} \quad x_{n4}]$ $y_n = 3$ $\hat{y}_n = 1$

We need to find a function that maps *x* and *y* for any of them.

$$\hat{y}_i = f(x_i; \theta)$$

How do we ”learn” the parameters of this function?

We choose ones that makes the following quantity small:

$$\sum_{i=1}^{n} Cost(\hat{y}_i, y_i)$$

Supervised Learning – Linear Softmax

Training Data

inputs

targets / labels / ground truth

$x_1 = [x_{11} \quad x_{12} \quad x_{13} \quad x_{14}]$ $y_1 = 1$

$x_2 = [x_{21} \quad x_{22} \quad x_{23} \quad x_{24}]$ $y_2 = 2$

$x_3 = [x_{31} \quad x_{32} \quad x_{33} \quad x_{34}]$ $y_3 = 1$

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$x_n = [x_{n1} \quad x_{n2} \quad x_{n3} \quad x_{n4}]$ $y_n = 3$

Supervised Learning – Linear Softmax

Training Data

inputs

targets / labels / ground truth

predictions

$x_1 = [x_{11} \quad x_{12} \quad x_{13} \quad x_{14}]$ $y_1 = [1 \quad 0 \quad 0]$ $\hat{y}_1 = [0.85 \quad 0.10 \quad 0.05]$

$x_2 = [x_{21} \quad x_{22} \quad x_{23} \quad x_{24}]$ $y_2 = [0 \quad 1 \quad 0]$ $\hat{y}_2 = [0.20 \quad 0.70 \quad 0.10]$

$x_3 = [x_{31} \quad x_{32} \quad x_{33} \quad x_{34}]$ $y_3 = [1 \quad 0 \quad 0]$ $\hat{y}_3 = [0.40 \quad 0.45 \quad 0.15]$

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$x_n = [x_{n1} \quad x_{n2} \quad x_{n3} \quad x_{n4}]$ $y_n = [0 \quad 0 \quad 1]$ $\hat{y}_n = [0.40 \quad 0.25 \quad 0.35]$

Supervised Learning – Linear Softmax

$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}]$ $\quad y_i = [1 \quad 0 \quad 0]$ $\quad \hat{y}_i = [f_1 \quad f_2 \quad f_3]$

$$a_1 = w_{11}x_{i1} + w_{12}x_{i2} + w_{13}x_{i3} + w_{14}x_{i4} + b_c$$

$$a_2 = w_{21}x_{i1} + w_{22}x_{i2} + w_{23}x_{i3} + w_{24}x_{i4} + b_d$$

$$a_3 = w_{31}x_{i1} + w_{32}x_{i2} + w_{33}x_{i3} + w_{34}x_{i4} + b_b$$

$$f_1 = e^{a_1}/(e^{a_1}+e^{a_2} + e^{a_3})$$

$$f_2 = e^{a_2}/(e^{a_1}+e^{a_2} + e^{a_3})$$

$$f_3 = e^{a_3}/(e^{a_1}+e^{a_2} + e^{a_3})$$

How do we find a good w and b?

$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}]$ $\quad y_i = [1 \quad 0 \quad 0]$ $\quad \hat{y}_i = [f_1(w,b) \quad f_2(w,b) \quad f_3(w,b)]$

We need to find w, and b that minimize the following:

$$L(w,b) = \sum_{i=1}^{n}\sum_{j=1}^{3} -y_{i,j}\log(\hat{y}_{i,j}) \quad = \sum_{i=1}^{n} -\log(\hat{y}_{i,label}) \quad = \sum_{i=1}^{n} -\log f_{i,label}(w,b)$$

Why?

Computing Analytic Gradients

This is what we have:

$$\ell(W,b) = -\log(\hat{y}_{label}(W,b)) = -\log\left(\frac{\exp(a_{label}(W,b))}{\sum_{k=1}^{3}\exp(a_k(W,b))}\right)$$

Computing Analytic Gradients

This is what we have:

$$\ell(W,b) = -\log(\hat{y}_{label}(W,b)) = -\log\left(\frac{\exp(a_{label}(W,b))}{\sum_{k=1}^{3}\exp(a_k(W,b))}\right)$$

$$\ell = -\log\left(\frac{\exp(a_{label})}{\sum_{k=1}^{3}\exp(a_k)}\right)$$

Reminder: $a_i = (w_{i,1}x_1 + w_{i,2}x_2 + w_{i,3}x_3 + w_{i,4}x_4) + b_i$

Computing Analytic Gradients

This is what we have:

$$\ell = -\log\left(\frac{\exp(a_{label})}{\sum_{k=1}^{3}\exp(a_k)}\right)$$

Computing Analytic Gradients

This is what we have:

$$\ell = -\log\left(\frac{\exp(a_{label})}{\sum_{k=1}^{3}\exp(a_k)}\right)$$

This is what we need:

$\frac{\partial \ell}{\partial w_{ij}}$ for each w_{ij}

$\frac{\partial \ell}{\partial b_i}$ for each b_i

Computing Analytic Gradients

This is what we have:

$$\ell = -\log\left(\frac{\exp(a_{label})}{\sum_{k=1}^{3}\exp(a_k)}\right)$$

Step 1: Chain Rule of Calculus

$$\frac{\partial \ell}{\partial w_{ij}} = \frac{\partial \ell}{\partial a_i}\frac{\partial a_i}{\partial w_{ij}} \qquad \frac{\partial \ell}{\partial b_i} = \frac{\partial \ell}{\partial a_i}\frac{\partial a_i}{\partial b_i}$$

Computing Analytic Gradients

This is what we have:

$$\ell = -\log\left(\frac{\exp(a_{label})}{\sum_{k=1}^{3}\exp(a_k)}\right)$$

Step 1: Chain Rule of Calculus

Let's do these first

$$\frac{\partial \ell}{\partial w_{ij}} = \frac{\partial \ell}{\partial a_i}\boxed{\frac{\partial a_i}{\partial w_{ij}}} \qquad \frac{\partial \ell}{\partial b_i} = \frac{\partial \ell}{\partial a_i}\boxed{\frac{\partial a_i}{\partial b_i}}$$

Computing Analytic Gradients

$$\frac{\partial a_i}{\partial w_{ij}}$$

$$\frac{\partial a_i}{\partial b_i}$$

$$a_i = (w_{i,1}x_1 + w_{i,2}x_2 + w_{i,3}x_3 + w_{i,4}x_4) + b_i$$

$$\frac{\partial a_i}{\partial w_{i,3}} = \frac{\partial}{\partial w_{i,3}}(w_{i,1}x_1 + w_{i,2}x_2 + w_{i,3}x_3 + w_{i,4}x_4) + b_i$$

$$\frac{\partial a_i}{\partial w_{i,3}} = x_3$$

$$\frac{\partial a_i}{\partial w_{i,j}} = x_j$$

Computing Analytic Gradients

$$\frac{\partial a_i}{\partial w_{i,j}} = x_j$$

$$\frac{\partial a_i}{\partial b_i}$$

$$a_i = (w_{i,1}x_1 + w_{i,2}x_2 + w_{i,3}x_3 + w_{i,4}x_4) + b_i$$

$$\frac{\partial a_i}{\partial b_i} = \frac{\partial}{\partial b_i}(w_{i,1}x_1 + w_{i,2}x_2 + w_{i,3}x_3 + w_{i,4}x_4) + b_i$$

$$\frac{\partial a_i}{\partial b_i} = 1$$

Computing Analytic Gradients

$$\frac{\partial a_i}{\partial w_{i,j}} = x_j$$

$$\frac{\partial a_i}{\partial b_i} = 1$$

Computing Analytic Gradients

This is what we have:

$$\ell = -\log\left(\frac{\exp(a_{label})}{\sum_{k=1}^{3}\exp(a_k)}\right)$$

Step 1: Chain Rule of Calculus

Now let's do this one (same for both!)

$$\frac{\partial \ell}{\partial w_{ij}} = \boxed{\frac{\partial \ell}{\partial a_i}}\frac{\partial a_i}{\partial w_{ij}} \qquad \frac{\partial \ell}{\partial b_i} = \boxed{\frac{\partial \ell}{\partial a_i}}\frac{\partial a_i}{\partial b_i}$$

Computing Analytic Gradients

$$\frac{\partial \ell}{\partial a_i} = \frac{\partial}{\partial a_i}\left[-\log\left(\frac{\exp(a_{label})}{\sum_{k=1}^{3}\exp(a_k)}\right)\right]$$

$$= \frac{\partial}{\partial a_i}\left[\log\left(\sum_{k=1}^{3}\exp(a_k)\right) - a_{label}\right]$$

In our cat, dog, bear classification example: i = {1, 2, 3}

Computing Analytic Gradients

$$\frac{\partial \ell}{\partial a_i} = \frac{\partial}{\partial a_i}\left[-\log\left(\frac{\exp(a_{label})}{\sum_{k=1}^{3}\exp(a_k)}\right)\right]$$

$$= \frac{\partial}{\partial a_i}\left[\log\left(\sum_{k=1}^{3}\exp(a_k)\right) - a_{label}\right]$$

In our cat, dog, bear classification example: i = {1, 2, 3}

Let's say: label = 2

We need: $\frac{\partial \ell}{\partial a_1}$ $\frac{\partial \ell}{\partial a_2}$ $\frac{\partial \ell}{\partial a_3}$

Computing Analytic Gradients

$$= \frac{\partial}{\partial a_i}\left[\log\left(\sum_{k=1}^{3}\exp(a_k)\right) - a_{label}\right]$$

$\frac{\partial \ell}{\partial a_1}$ $\frac{\partial \ell}{\partial a_3}$

when $i \neq label$:

$$\frac{\partial \ell}{\partial a_i} = \frac{\partial}{\partial a_i}\left[\log\left(\sum_{k=1}^{3}\exp(a_k)\right) - a_{label}\right]$$

$$\frac{\partial \ell}{\partial a_i} = \frac{\partial}{\partial a_i}\log\left(\sum_{k=1}^{3}\exp(a_k)\right)$$

$$\frac{\partial \ell}{\partial a_i} = \left(\frac{1}{\sum_{k=1}^{3}\exp(a_k)}\right)\left(\frac{\partial}{\partial a_i}\sum_{k=1}^{3}\exp(a_k)\right)$$

$$\frac{\partial \ell}{\partial a_i} = \frac{\exp(a_i)}{\sum_{k=1}^{3}\exp(a_k)}$$

Supervised Learning – Linear Softmax

$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}]$ $\quad y_i = [1 \quad 0 \quad 0]$ $\quad \hat{y}_i = [f_1 \quad f_2 \quad f_3]$

$$a_1 = w_{11}x_{i1} + w_{12}x_{i2} + w_{13}x_{i3} + w_{14}x_{i4} + b_c$$

$$a_2 = w_{21}x_{i1} + w_{22}x_{i2} + w_{23}x_{i3} + w_{24}x_{i4} + b_d$$

$$a_3 = w_{31}x_{i1} + w_{32}x_{i2} + w_{33}x_{i3} + w_{34}x_{i4} + b_b$$

$$f_1 = e^{a_1}/(e^{a_1}+e^{a_2} + e^{a_3})$$

$$f_2 = e^{a_2}/(e^{a_1}+e^{a_2} + e^{a_3})$$

$$f_3 = e^{a_3}/(e^{a_1}+e^{a_2} + e^{a_3})$$

Computing Analytic Gradients

$$= \frac{\partial}{\partial a_i}\left[\log\left(\sum_{k=1}^{3}\exp(a_k)\right) - a_{label}\right]$$

$\frac{\partial \ell}{\partial a_1}$ $\frac{\partial \ell}{\partial a_3}$

when $i \neq label$:

$$\frac{\partial \ell}{\partial a_i} = \frac{\partial}{\partial a_i}\left[\log\left(\sum_{k=1}^{3}\exp(a_k)\right) - a_{label}\right]$$

$$\frac{\partial \ell}{\partial a_i} = \frac{\partial}{\partial a_i}\log\left(\sum_{k=1}^{3}\exp(a_k)\right)$$

$$\frac{\partial \ell}{\partial a_i} = \left(\frac{1}{\sum_{k=1}^{3}\exp(a_k)}\right)\left(\frac{\partial}{\partial a_i}\sum_{k=1}^{3}\exp(a_k)\right)$$

$$\frac{\partial \ell}{\partial a_i} = \frac{\exp(a_i)}{\sum_{k=1}^{3}\exp(a_k)}$$

Computing Analytic Gradients

$$= \frac{\partial}{\partial a_i}\left[\log\left(\sum_{k=1}^{3}\exp(a_k)\right) - a_{label}\right]$$

$\frac{\partial \ell}{\partial a_2}$

when $i = label$:

$$\frac{\partial \ell}{\partial a_{label}} = \frac{\partial}{\partial a_{label}}\left[\log\left(\sum_{k=1}^{3}\exp(a_k) - a_{label}\right)\right]$$

$$\frac{\partial \ell}{\partial a_{label}} = \frac{\partial}{\partial a_{label}}\log\left(\sum_{k=1}^{3}\exp(a_k)\right) - 1$$

$$\frac{\partial \ell}{\partial a_{label}} = \left(\frac{1}{\sum_{k=1}^{3}\exp(a_k)}\right)\left(\frac{\partial}{\partial a_{label}}\sum_{k=1}^{3}\exp(a_k)\right) - 1$$

$$\frac{\partial \ell}{\partial a_{label}} = \frac{\exp(a_{label})}{\sum_{k=1}^{3}\exp(a_k)} - 1$$

$\hat{y}_i - 1$

Computing Analytic Gradients

label = 1

$$\frac{\partial \ell}{\partial a_1} = \hat{y}_1 \qquad \frac{\partial \ell}{\partial a_2} = \hat{y}_2 - 1 \qquad \frac{\partial \ell}{\partial a_3} = \hat{y}_3$$

$$\frac{\partial \ell}{\partial a} = \begin{bmatrix} \frac{\partial \ell}{\partial a_1} \\ \frac{\partial \ell}{\partial a_2} \\ \frac{\partial \ell}{\partial a_3} \end{bmatrix} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 - 1 \\ \hat{y}_3 \end{bmatrix} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \hat{y} - y$$

$$\frac{\partial \ell}{\partial a_i} = \hat{y}_i - y_i$$

Computing Analytic Gradients

$$\frac{\partial \ell}{\partial w_{ij}} = \frac{\partial \ell}{\partial a_i} \frac{\partial a_i}{\partial w_{ij}}$$

$$\frac{\partial \ell}{\partial b_i} = \frac{\partial \ell}{\partial a_i} \frac{\partial a_i}{\partial b_i}$$

$$\frac{\partial a_i}{\partial w_{i,j}} = x_j$$

$$\frac{\partial a_i}{\partial b_i} = 1$$

$$\frac{\partial \ell}{\partial a_i} = \hat{y}_i - y_i$$

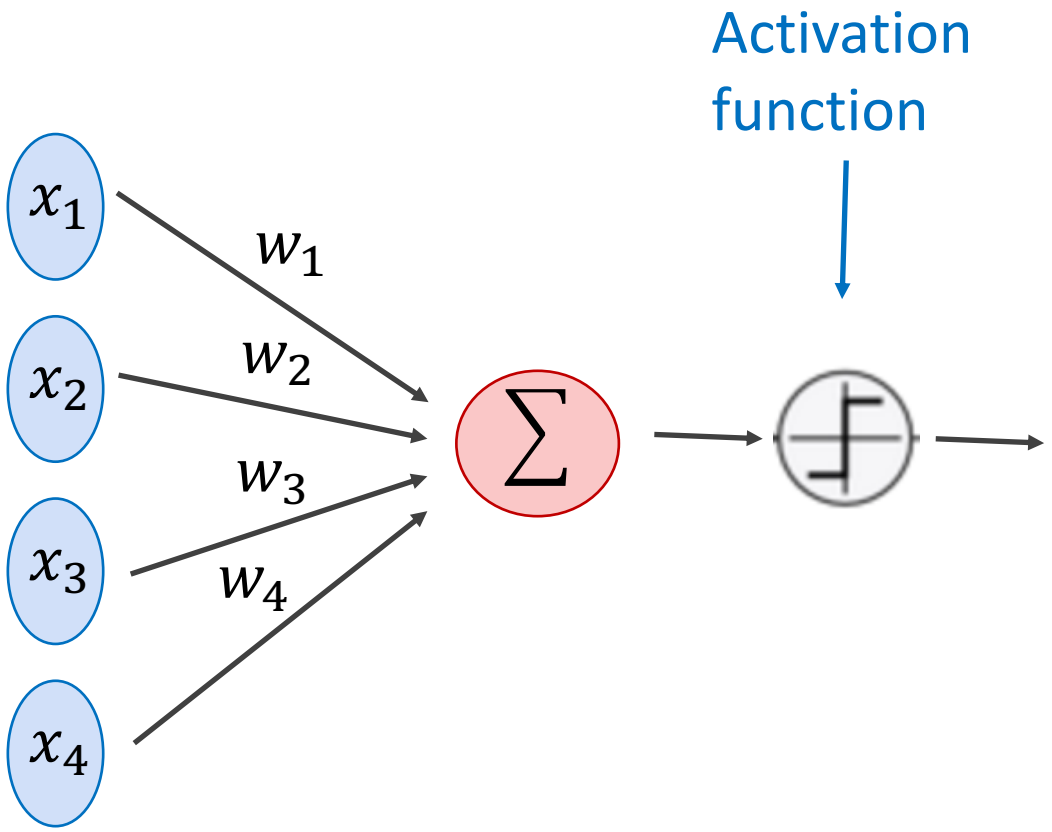
$$\boxed{\frac{\partial \ell}{\partial w_{i,j}} = (\hat{y}_i - y_i)x_j}$$

$$\boxed{\frac{\partial \ell}{\partial b_i} = (\hat{y}_i - y_i)}$$

Perceptron Model

Frank Rosenblatt (1957) - Cornell University

$$f(x) = \begin{cases} 1, & \text{if } \sum_{i=0}^{n} w_i x_i + b > 0 \\ 0, & \text{otherwise} \end{cases}$$



More: https://en.wikipedia.org/wiki/Perceptron

Perceptron Model

Frank Rosenblatt (1957) - Cornell University

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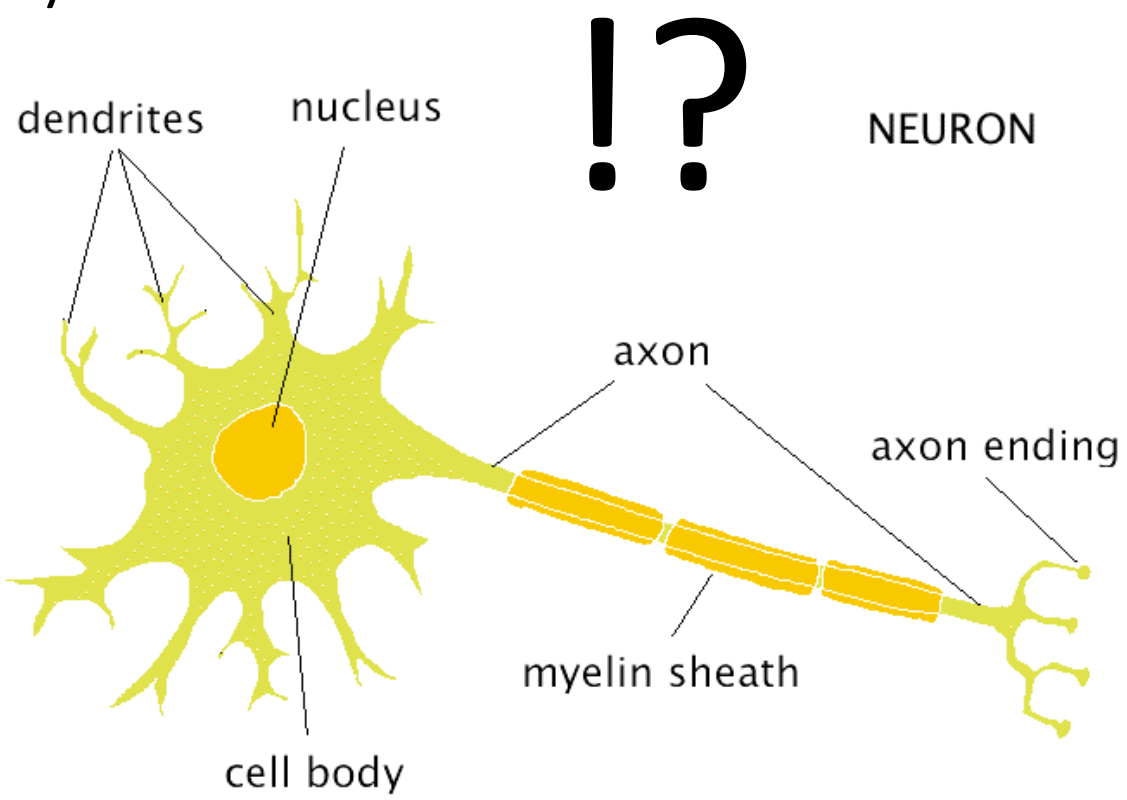
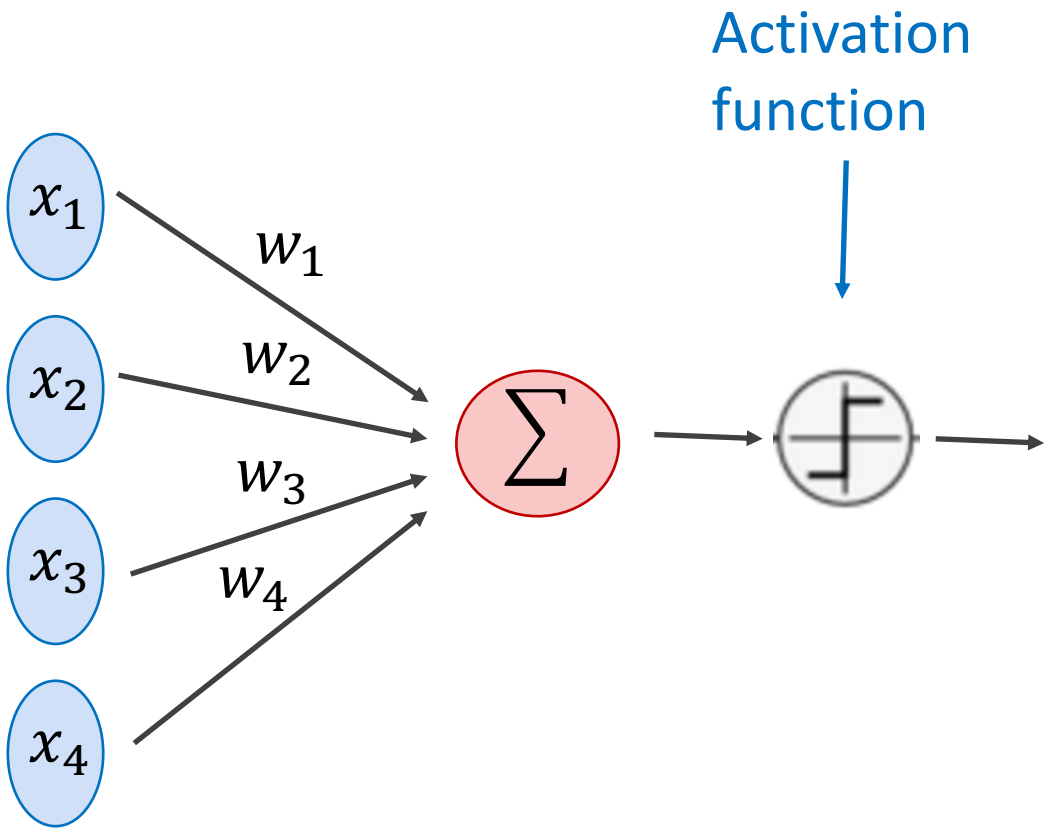


More: https://en.wikipedia.org/wiki/Perceptron

Perceptron Model

Frank Rosenblatt (1957) - Cornell University

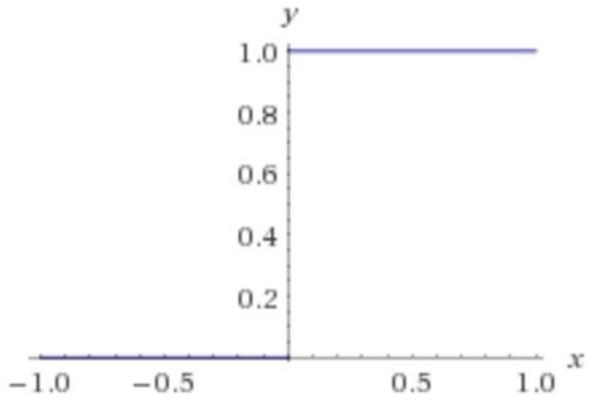
$$f(x) = \begin{cases} 1, & \text{if } \sum_{i=0}^{n} w_i x_i + b > 0 \\ 0, & \text{otherwise} \end{cases}$$



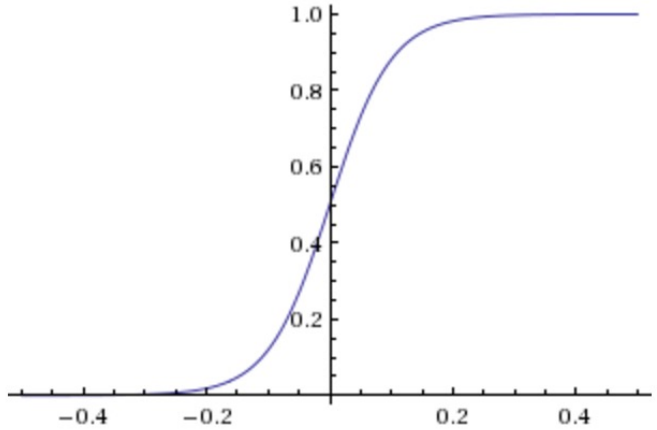
More: https://en.wikipedia.org/wiki/Perceptron

Activation Functions

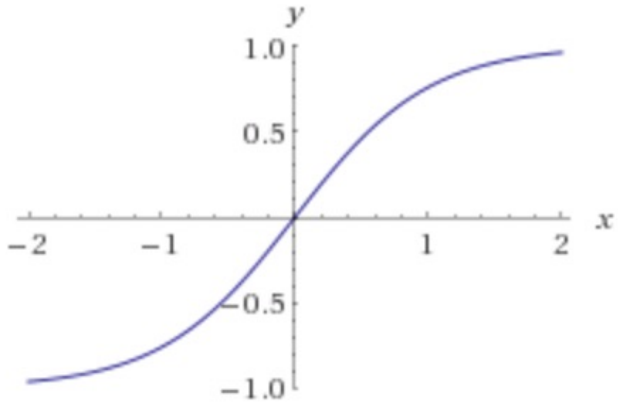
Step(x)



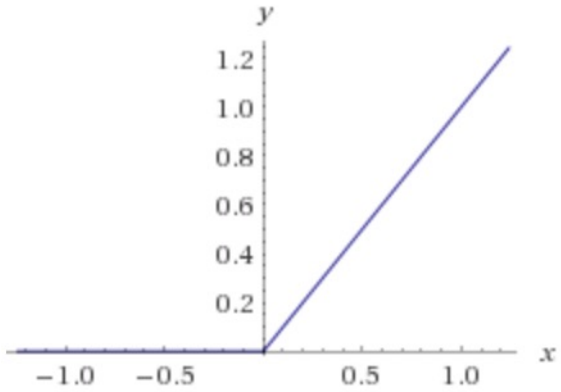
Sigmoid(x)



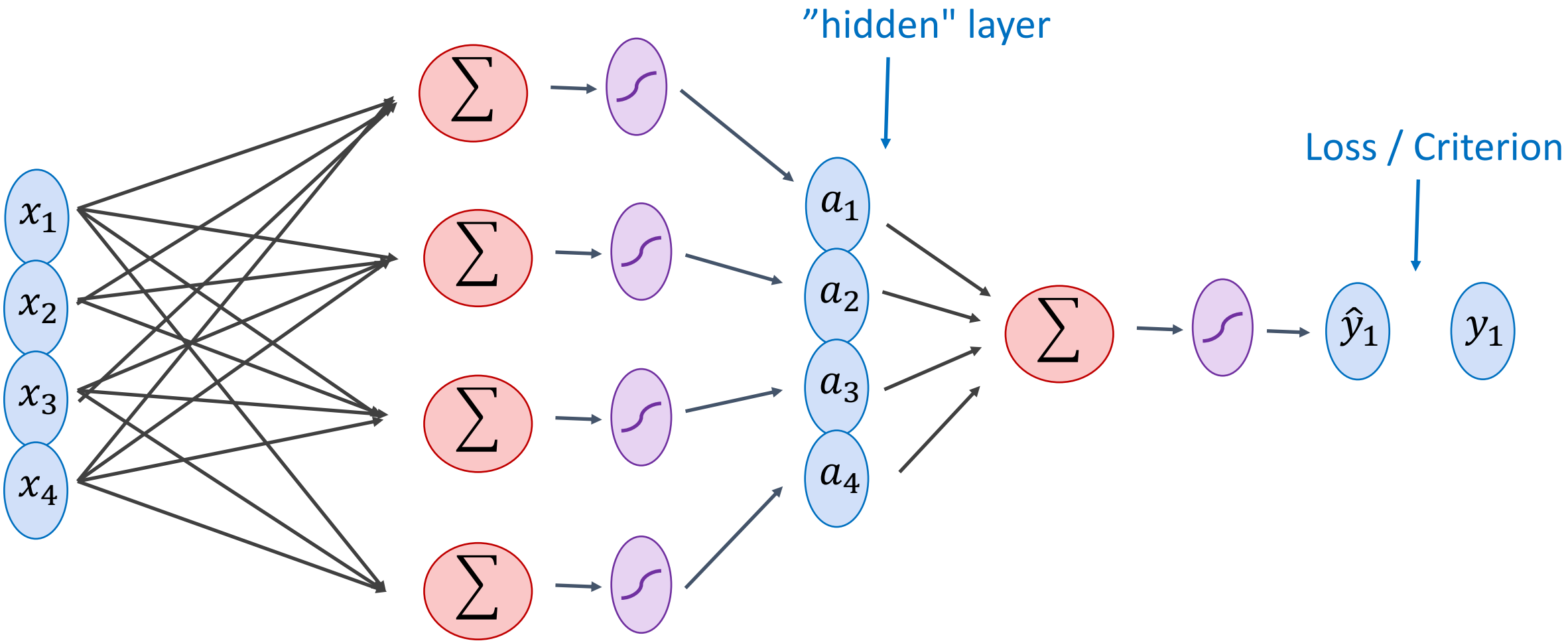
Tanh(x)



ReLU(x) = max(0, x)



Two-layer Multi-layer Perceptron (MLP)



Linear Softmax

$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}]$ $\qquad y_i = [1 \quad 0 \quad 0]$ $\qquad \hat{y}_i = [f_c \quad f_d \quad f_b]$

$$g_c = w_{c1}x_{i1} + w_{c2}x_{i2} + w_{c3}x_{i3} + w_{c4}x_{i4} + b_c$$

$$g_d = w_{d1}x_{i1} + w_{d2}x_{i2} + w_{d3}x_{i3} + w_{d4}x_{i4} + b_d$$

$$g_b = w_{b1}x_{i1} + w_{b2}x_{i2} + w_{b3}x_{i3} + w_{b4}x_{i4} + b_b$$

$$f_c = e^{g_c}/(e^{g_c}+e^{g_d} + e^{g_b})$$

$$f_d = e^{g_d}/(e^{g_c}+e^{g_d} + e^{g_b})$$

$$f_b = e^{g_b}/(e^{g_c}+e^{g_d} + e^{g_b})$$

Linear Softmax

$$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}] \qquad y_i = [1 \quad 0 \quad 0] \qquad \hat{y}_i = [f_c \quad f_d \quad f_b]$$

$$g_c = w_{c1}x_{i1} + w_{c2}x_{i2} + w_{c3}x_{i3} + w_{c4}x_{i4} + b_c$$

$$g_d = w_{d1}x_{i1} + w_{d2}x_{i2} + w_{d3}x_{i3} + w_{d4}x_{i4} + b_d$$

$$g_b = w_{b1}x_{i1} + w_{b2}x_{i2} + w_{b3}x_{i3} + w_{b4}x_{i4} + b_b$$

$$w = \begin{bmatrix} w_{c1} & w_{c2} & w_{c3} & w_{c4} \\ w_{d1} & w_{d2} & w_{d3} & w_{d4} \\ w_{b1} & w_{b2} & w_{b3} & w_{b4} \end{bmatrix}$$

$$b = [b_c \quad b_d \quad b_b]$$

$$f_c = e^{g_c}/(e^{g_c}+e^{g_d}+e^{g_b})$$

$$f_d = e^{g_d}/(e^{g_c}+e^{g_d}+e^{g_b})$$

$$f_b = e^{g_b}/(e^{g_c}+e^{g_d}+e^{g_b})$$

Linear Softmax

$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}]$

$y_i = [1 \quad 0 \quad 0]$

$\hat{y}_i = [f_c \quad f_d \quad f_b]$

$$g = wx^T + b^T$$

$$w = \begin{bmatrix} w_{c1} & w_{c2} & w_{c3} & w_{c4} \\ w_{d1} & w_{d2} & w_{d3} & w_{d4} \\ w_{b1} & w_{b2} & w_{b3} & w_{b4} \end{bmatrix}$$

$b = [b_c \quad b_d \quad b_b]$

$$f_c = e^{g_c}/(e^{g_c}+e^{g_d}+e^{g_b})$$

$$f_d = e^{g_d}/(e^{g_c}+e^{g_d}+e^{g_b})$$

$$f_b = e^{g_b}/(e^{g_c}+e^{g_d}+e^{g_b})$$

Linear Softmax

$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}]$

$y_i = [1 \quad 0 \quad 0]$

$\hat{y}_i = [f_c \quad f_d \quad f_b]$

$$g = wx^T + b^T$$

$$w = \begin{bmatrix} w_{c1} & w_{c2} & w_{c3} & w_{c4} \\ w_{d1} & w_{d2} & w_{d3} & w_{d4} \\ w_{b1} & w_{b2} & w_{b3} & w_{b4} \end{bmatrix}$$

$$b = [b_c \quad b_d \quad b_b]$$

$$f = softmax(g)$$

Linear Softmax

$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}]$ $y_i = [1 \quad 0 \quad 0]$ $\hat{y}_i = [f_c \quad f_d \quad f_b]$

$$f = softmax(wx^T + b^T)$$

Two-layer MLP + Softmax

$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}]$ $y_i = [1 \quad 0 \quad 0]$ $\hat{y}_i = [f_c \quad f_d \quad f_b]$

$$a_1 = sigmoid(w_{[1]}x^T + b_{[1]}^T)$$

$$f = softmax(w_{[2]}{a_1}^T + b_{[2]}^T)$$

N-layer MLP + Softmax

$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}]$ $y_i = [1 \quad 0 \quad 0]$ $\hat{y}_i = [f_c \quad f_d \quad f_b]$

$$a_1 = sigmoid(w_{[1]}x^T + b_{[1]}^T)$$

$$a_2 = sigmoid(w_{[2]}a_1^T + b_{[2]}^T)$$

...

$$a_k = sigmoid(w_{[k]}a_{k-1}^T + b_{[k]}^T)$$

...

$$f = softmax(w_{[n]}a_{n-1}^T + b_{[n]}^T)$$

How to train the parameters?

$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}]$ $y_i = [1 \quad 0 \quad 0]$ $\hat{y}_i = [f_c \quad f_d \quad f_b]$

$$a_1 = sigmoid(w_{[1]}x^T + b_{[1]}^T)$$

$$a_2 = sigmoid(w_{[2]}a_1^T + b_{[2]}^T)$$

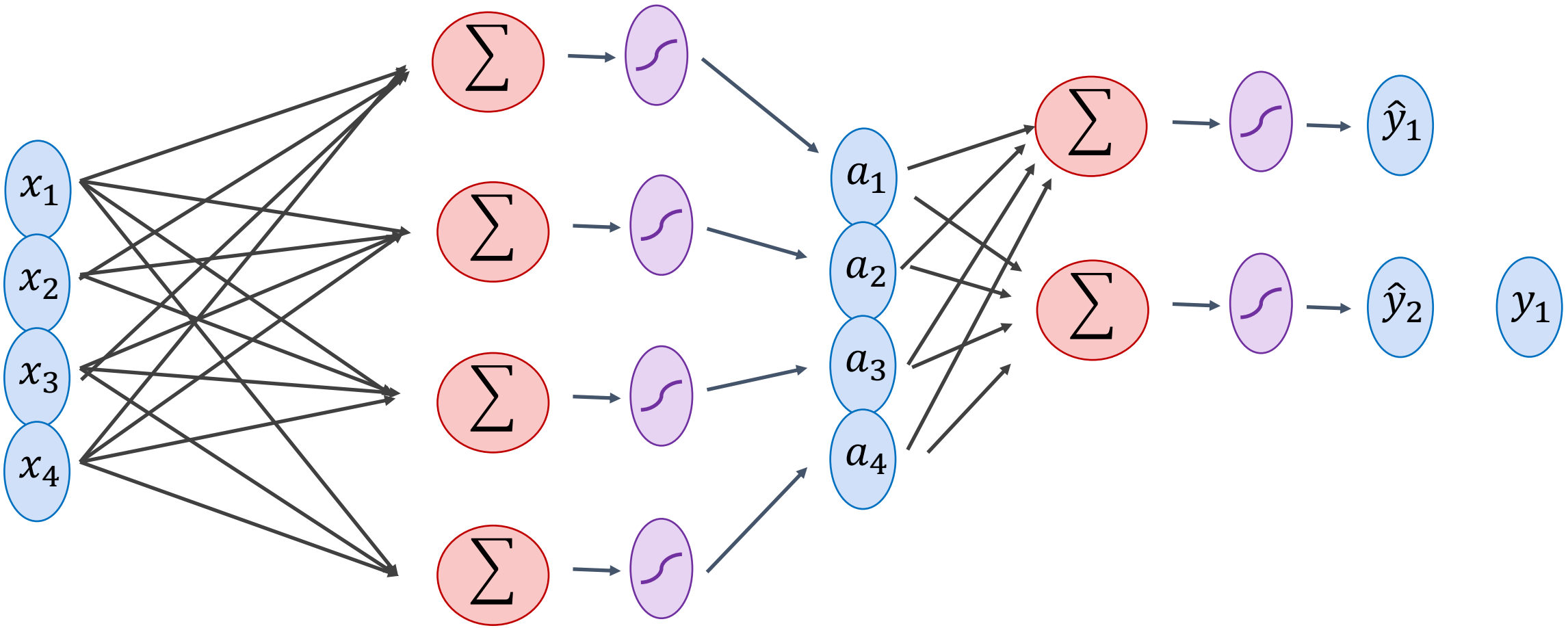
...

$$a_k = sigmoid(w_{[k]}a_{k-1}^T + b_{[k]}^T)$$

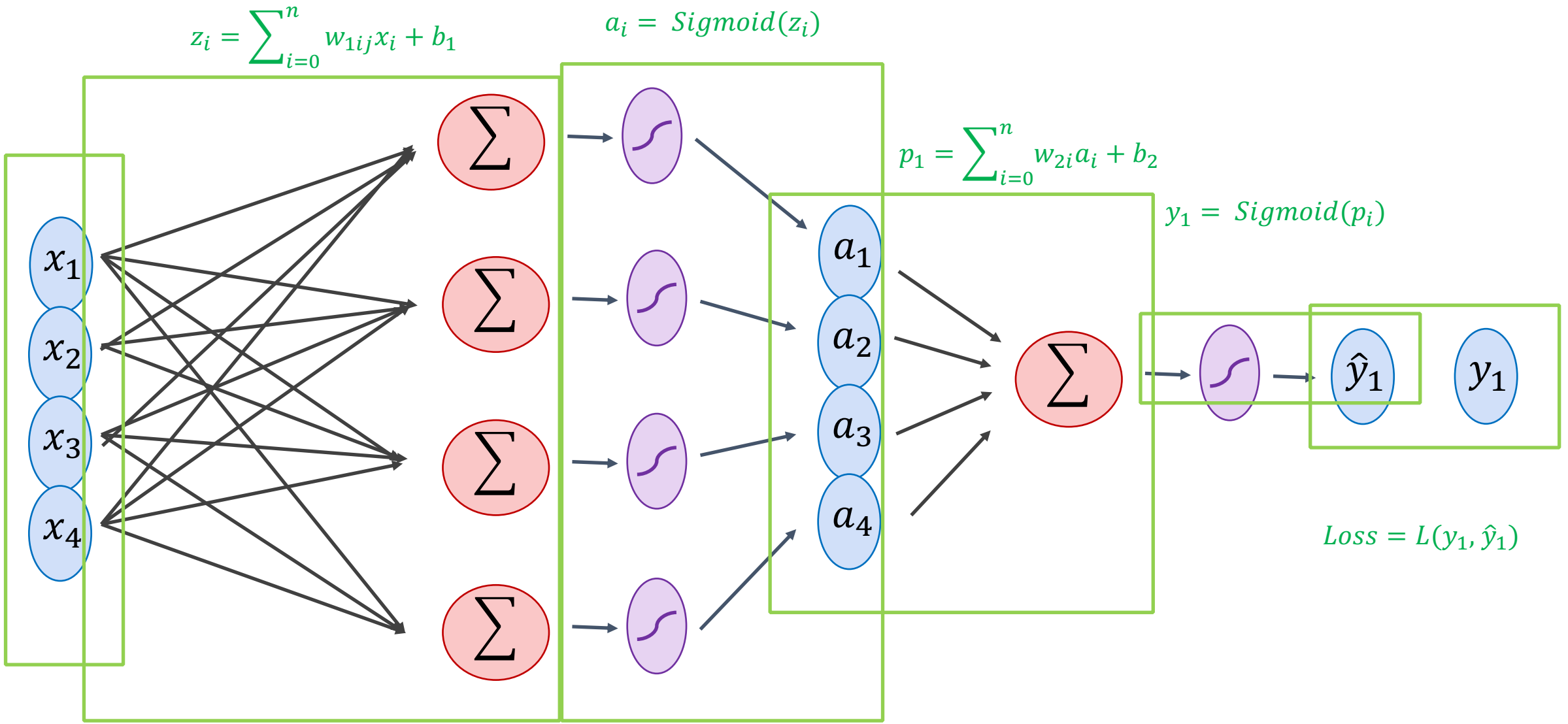
...

$$f = softmax(w_{[n]}a_{n-1}^T + b_{[n]}^T)$$

Forward pass (Forward-propagation)



Forward pass (Forward-propagation)



How to train the parameters?

$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}]$ $y_i = [1 \quad 0 \quad 0]$ $\hat{y}_i = [f_c \quad f_d \quad f_b]$

$$a_1 = sigmoid(w_{[1]}x^T + b_{[1]}^T)$$

$$a_2 = sigmoid(w_{[2]}a_1^T + b_{[2]}^T)$$

...

$$a_k = sigmoid(w_{[k]}a_{k-1}^T + b_{[i]}^T)$$

...

$$f = softmax(w_{[n]}a_{n-1}^T + b_{[n]}^T)$$

We can still use SGD

We need!

$$\frac{\partial l}{\partial w_{[k]ij}} \qquad \frac{\partial l}{\partial b_{[k]i}}$$

How to train the parameters?

$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}]$ $y_i = [1 \quad 0 \quad 0]$ $\hat{y}_i = [f_c \quad f_d \quad f_b]$

$$a_1 = sigmoid(w_{[1]}x^T + b_{[1]}^T)$$

$$a_2 = sigmoid(w_{[2]}a_1^T + b_{[2]}^T)$$

...

$$a_i = sigmoid(w_{[k]}a_{k-1}^T + b_{[k]}^T)$$

...

$$f = softmax(w_{[n]}a_{n-1}^T + b_{[n]}^T)$$

$$l = loss(f, y)$$

We can still use SGD

We need!

$$\frac{\partial l}{\partial w_{[k]ij}} \qquad \frac{\partial l}{\partial b_{[k]i}}$$

How to train the parameters?

$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}]$ $y_i = [1 \quad 0 \quad 0]$ $\hat{y}_i = [f_c \quad f_d \quad f_b]$

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$$\frac{\partial l}{\partial w_{[k]ij}} \qquad \frac{\partial l}{\partial b_{[k]i}}$$

How to train the parameters?

$x_i = [x_{i1} \quad x_{i2} \quad x_{i3} \quad x_{i4}]$ $\quad y_i = [1 \quad 0 \quad 0]$ $\quad \hat{y}_i = [f_c \quad f_d \quad f_b]$

$$a_1 = sigmoid(w_{[1]}x^T + b_{[1]}^T)$$

$$a_2 = sigmoid(w_{[2]}a_1^T + b_{[2]}^T)$$

...

$$a_i = sigmoid(w_{[k]}a_{k-1}^T + b_{[k]}^T)$$

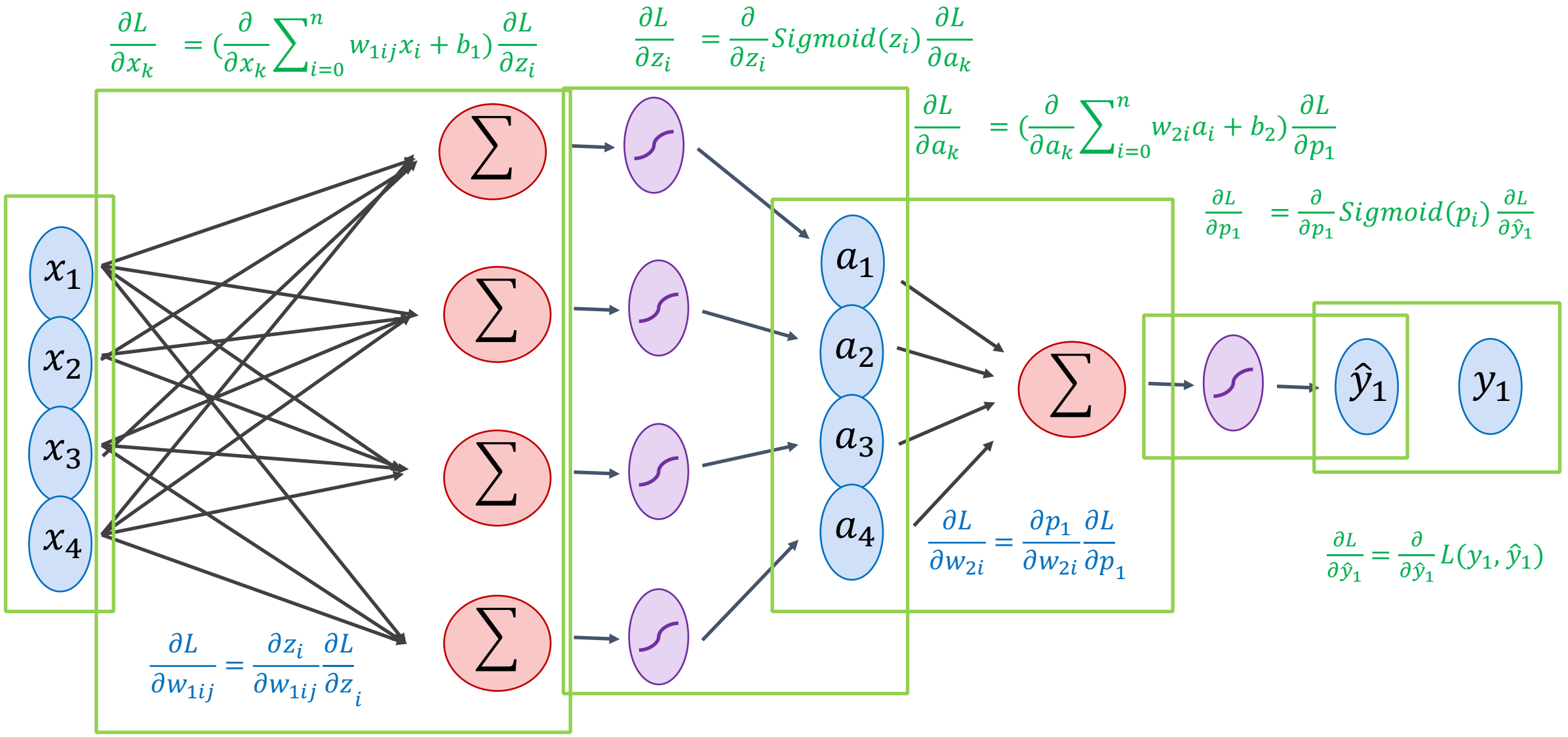
...

$$f = softmax(w_{[n]}a_{n-1}^T + b_{[n]}^T)$$

$$l = loss(f, y)$$

$$\frac{\partial l}{\partial w_{[k]ij}} = \frac{\partial l}{\partial a_{n-1}} \frac{\partial a_{n-1}}{\partial a_{n-2}} \cdots \frac{\partial a_{k-2}}{\partial a_{k-1}} \frac{\partial a_{k-1}}{\partial w_{[k]ij}}$$

Backward pass (Back-propagation)



Questions?