# Expected length of a chain of failure events

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Consider an event with two possible outcomes, “success” and “failure”. Now consider a chain of *n+1* of these events where there are *n* failures followed by a single success.

Fail 🡪 Fail 🡪 Fail 🡪 … 🡪 Fail 🡪 Success

***What is the expected length of such a chain of events, as measured by the number of failures?***

## Derivation:

Let

*ps* = the probability of success = *1- pf*

*pf* = the probability of failure = *1- ps*

The probability of a chain of length *n* is thus

*Pn = ps ∙ pfn = (1- pf) ∙ pfn*

The total probability for a chain of any length is the sum over all possible chains and is thus one, as expected:

The expected length of a chain is the weighted sum of the individual probabilities:

It will be easier to solve for the sum separately:

Solving for the sum:

The expected length is thus

The expected length is the ratio of the failure and success probabilities. The more likely the failure, the longer the expected length of the chain will be on average and vice versa. For the degenerate case of equal probabilities, such as in coin tosses, the expected length is 1; that is, when tossing coins for heads up, one can expect on average to experience one tails and then a heads.