Automata B-Exam
May 1996

There are 3 questions worth 10 points each for a total of 30 points.

1. Let $L_n = \{x \in \{0,1\}^* \mid x \text{ is a binary number that is multiple of } n\}$. Prove that for each $n \geq 1$ the language L_n is regular.

Hint: Prove that there are at most n equivalence classes and apply Myhill–Nerode's Theorem.

2. Let L be a language. Prove that L is recursively enumerable if a recursive language D exists such that $L = \{x \mid \exists y \text{ such that } xy \in D\}$.

3. A *spanning tree* of a graph H is any subgraph of H which is a tree and contains all vertices of H. The *spanning tree problem* is to decide, given a graph G and a tree T, whether T is isomorphic to any spanning tree of G. Prove that the Hamiltonian cycle problem is log-space reducible to the spanning tree problem.