# Automata B-Exam
# January 1996

There are 4 questions worth 10 points each for a total of 40 points.

1. For any two integers $p$ and $q$ denote

$$A_{pq} = \{x \mid x = p + nq,\ x \geq 0,\ n - \text{integer}\}.$$

   Show that the language

$$R = \{a^i \mid i \in A_{pq}\}$$

   is regular.

2. Let

$$L = \{a^n(a+b)^{n+m}b^m \mid n\text{-odd},\ m\text{-even}\}.$$

   Is $L$ a CFL? Justify your answer.

3. Let the language $L$ be recursively enumerable but not recursive. Show that for any Turing machine $M$ that accepts $L$ there are infinitely many strings on which $M$ does not halt.

4. Let $L_1$ and $L_2$ be languages.

   - Suppose that there is a $2^n$ time-bounded reduction of $L_1$ to $L_2$, and $L_2$ is in DTIME($2^n$). What can you conclude about $L_1$?
   - Suppose that there is a log-space reduction of $L_1$ to $L_2$, and $L_2$ is the *graph reachability problem*. What can you conclude about $L_1$?