1. Give an asymptotically tight solution (big θ) for each of the following recurrences. You may assume that $T(1) = 1$. You need not prove your solution correct.

   (a)

   $$T(n) = T(\frac{1}{2}n) + T(\frac{1}{4}n) + T(\frac{1}{6}n) + n.$$

   (b)

   $$T(n) = 9T(\frac{1}{3}n) + 2n^2.$$

   (c)

   $$T(n) = T(\sqrt{n}) + 1.$$

2. Given a sequence a_1, a_2, ..., a_n and a permutation $\pi(1)$, $\pi(2)$, ..., $\pi(n)$, give an $O(n)$ algorithm to rearrange the sequence in place into the order $a_{\pi(1)}$, $a_{\pi(2)}$, ..., $a_{\pi(n)}$. Your algorithm may use at most $n + O(1)$ extra bits of space. (Note that this restriction means you cannot copy into an auxiliary array or use recursion.)

3. State the definition of a binomial tree B_k. Prove that a binomial tree B_k has $C(i, k) = \frac{k!}{i!(k-i)!}$ nodes at depth i.

4. Given a directed, acyclic graph $G = (V, E)$ and a pair of vertices v_1 and v_2 in V, describe an $O(V + E)$ algorithm for computing the total number of distinct paths from v_1 to v_2 in G.